

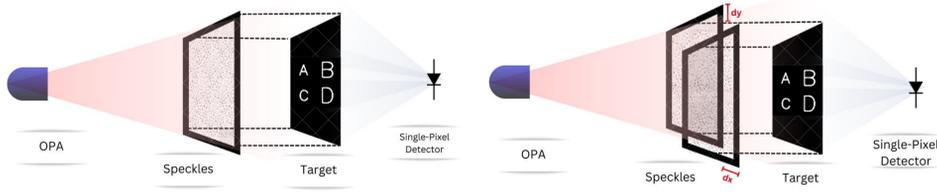
## Motivation

Modern diagnostics increasingly rely on imaging technologies that are **compact, high resolution**, and ideally **integrated into photonic circuits**. Our long term research aims to **improve healthcare diagnostics** using **Photonic Integrated Circuits (PICs)** enhanced with **Optical Feedback Interferometry (OFI)** and **Compressed Sensing (CS)** techniques. This study is part of the MIRABILIS project (PRIN 2022), focused on advancing **Mid Infrared (MIR) imaging systems for early disease detection using single pixel imaging (SPI)**.

**CS enables SPI to work efficiently** by reconstructing the image from far fewer measurements, exploiting sparsity of the target. In traditional systems, either scan one pixel at a time (slow) or use a big sensor array (expensive). We **simulate CS**, which allows us to capture an entire image using only a single pixel detector, by illuminating the object with a series of **pseudo random patterns**. In our simulations, these pseudo-random patterns represent the mathematical requirement of CS. **In a physical system**, the same role is played by **speckle patterns**, which are the optical realization of pseudo random illumination fields.

Random speckle patterns play a vital role in CS. In our exploration, we consider two potential solution for their generation: A typical **device for speckle pattern generation is OPA**, which can be controlled not only to create a focused beam (e.g., for raster scanning), but also, with proper control signals, **to generate pseudo random patterns** whose characteristic size depends mainly on the number of branches. While the reproducibility of these patterns is generally very good, it can still be **influenced by temperature variations and mechanical displacements of the experimental setup**. For this reason, **we first investigate the effect of such uncertainties through simulation, before moving to experimental validation**.

An additional solution can be represented by a nanocomposite combining electro-spun nanofiber tissue and UV curable resin on a glass plate, with a total area much larger than the target image. In this way, it is possible to **generate the various uncorrelated patterns simply by translating the plate in the x and y directions**, using different uncorrelated areas. In this scenario, a critical aspect is the proper alignment of the plate in the expected position.

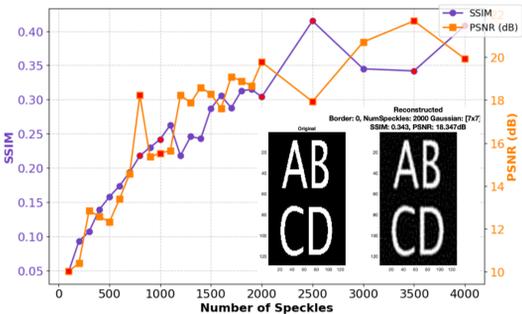


### (a) Perfect alignment ( $R = 0$ ):

In the ideal case, the speckle patterns projected on the target are exactly the expected ones, used in the reconstruction matrix. The system works as expected, and the image can be reliably reconstructed.

### (b) Misalignment ( $R = 1, 2, 4, 6$ ):

When the speckle patterns are shifted in the x- or y-direction (by 2, 4, or 6 pixels), the projected patterns no longer match the reconstruction matrix. Even small misalignments cause degradation in the reconstructed image. **Our work quantifies this sensitivity and defines the tolerance limits for reliable reconstruction.**



Effect of number of speckle patterns on SSIM and PSNR for  $R = 0$  (perfect alignment). Inset: target image (left) and reconstructed result (right) with 2000 patterns (SSIM = 0.343, PSNR = 18.3 dB).

Our image has 16,384 pixels ( $128 \times 128$ ), so raster scanning would require the same number of patterns. In practice, we use far fewer, up to 4000 patterns ( $\approx 25\%$  of full sampling). As  $K$  increases, reconstruction quality improves: around 2000–3000 patterns ( $\approx 12\text{--}18\%$  of full sampling) the image is already readable, with SSIM  $\approx 0.34$  and PSNR  $\approx 18$  dB. Beyond that, improvements plateau, showing that CS enables good reconstructions from only a fraction of the measurements.

## Novel contributions

1. We have shown through numerical calculations that spatial misalignment can significantly amplify inaccuracies in CS. As the alignment error increases, speckle shifts cause significant structural decorrelation, harming reconstruction fidelity.
2. We demonstrate that even minor spatial misalignments can considerably influence the quality of reconstruction, emphasising a definitive correlation between the misalignment range ( $R$ ) and the reduction in SSIM/PSNR scores.
3. When misalignment is present ( $R > 0$ ), adding more than 1000 speckle patterns actually starts to lower the image quality due to noise accumulation.
4. We demonstrate that readable reconstructions can be achieved with only 2000–3000 speckles, the best in the image with  $R > 0$  is around 1000 ( $\sim 12\text{--}18\%$  of full sampling for  $128 \times 128$  images), proving the efficiency of compressed sensing.

## Methodology

Our reconstruction pipeline is implemented in MATLAB, and we systematically investigate how the image quality degrades when the projected patterns are slightly misaligned, a crucial concern in practical optical systems like OPA-based speckle generators.

### Compressed Sensing

$$A_k = \sum_x \sum_y I_k(x,y) \cdot T(x,y) = \langle I_k, T \rangle, \quad k = 1, \dots, K$$

**Definitions:**

$A_k$ : scalar intensity measured by the single-pixel detector for the  $k$ -th pattern

$I_k(x,y)$ : illumination pattern at pixel  $(x,y)$

$T(x,y)$ : target image value at pixel  $(x,y)$

$N \times K$ : image resolution

$N^2$ : total number of pixels

$K$ : total number of projected speckle patterns ( $\leq N^2$ )

**Matrix formulation:**

$$A = I^T T$$

$I \in \mathbb{R}^{N^2 \times K}$ : columns  $I_k$  contain lexicographically ordered  $N \times N$  patterns

$T \in \mathbb{R}^{N^2 \times 1}$ : unknown target (flattened image)

$A \in \mathbb{R}^{K \times 1}$ : detector measurements

### Reconstruction (Moore–Penrose pseudoinverse):

$$\tilde{T} = I^+ A$$

Where  $I^+$  is the Moore–Penrose generalized inverse of  $I$ .

**Key difference:**

**SPI (full sampling):**  $K = N^2$ , exact reconstruction possible.

**CS-SPI (compressive):**  $K \ll N^2$ , reconstruction requires sparsity, solved via optimization. Works when system is overdetermined ( $K \geq N^2$ ) or noisy.

### 1. Illuminate the object

First, we illuminate the object with many known light patterns. We shine the object with many known patterns  $I_k(x,y)$ . Each pattern interacts with the target image  $T(x,y)$ .

### 2. Measure total light

Second, instead of recording a whole image, we only record a single number per pattern. A single-pixel detector collects just one number per pattern:  $A_k = \sum_x \sum_y I_k(x,y) T(x,y)$ . This is the inner product (overlap) between pattern and object.

### 3. Build measurement matrix

Third, we stack all those numbers and patterns into a matrix equation. This only tells us **how the detector values are generated** from the patterns and the target. It does **not tell us how to get back**  $T$  from the measurements  $A$ . Collect all patterns into matrix  $I(N^2 \times K)$ . Collect all detector values into vector  $A(K \times 1)$ . Compact model:  $A = I^T T$ .

### 4. Reconstruct image

Finally, we mathematically invert that equation to recover the image. Use Moore–Penrose pseudoinverse to estimate:  $\tilde{T} = I^+ A$ . Invert correlation between patterns and measurements.

## Evaluation Metrics

### Structural Similarity Index (SSIM):

Compares **structure** between reconstructed and original image.

Range:  $[0, 1] \rightarrow$  closer to 1 = more similar.

Sensitive to **contrast, luminance, and texture**.

It tells if the image "looks right" to the human eye.

$$SSIM(T, \tilde{T}) = \frac{(2\mu_T \mu_{\tilde{T}} + C_1)(2\sigma_{T\tilde{T}} + C_2)}{(\mu_T^2 + \mu_{\tilde{T}}^2 + C_1)(\sigma_T^2 + \sigma_{\tilde{T}}^2 + C_2)}$$

$\mu_T, \mu_{\tilde{T}}$ : mean intensity of original and reconstructed images

$\sigma_T^2, \sigma_{\tilde{T}}^2$ : variance of each image

$\sigma_{T\tilde{T}}$ : covariance between them

$C_1, C_2$ : small constants to stabilize division

### Peak Signal-to-Noise Ratio (PSNR):

Compares **pixel-level error** between reconstructed and original. Higher PSNR (in dB) = less error, better quality. Sensitive to overall noise and distortions.

$$PSNR = 10 \cdot \log_{10} \left( \frac{MAX_I^2}{MSE} \right)$$

Where

$$MSE = \frac{1}{N^2} \sum_{x,y} (T(x,y) - \tilde{T}(x,y))^2$$

$MAX_I$ : maximum possible pixel value (e.g., 255 for 8-bit images)

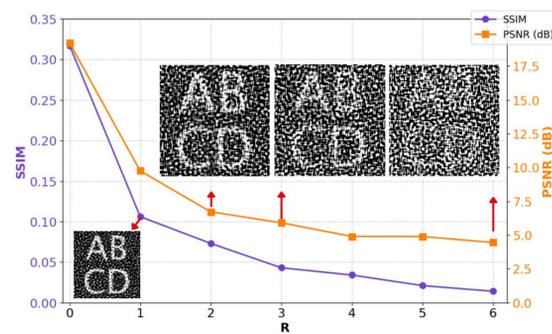
$T(x,y)$ : original target pixel value

$\tilde{T}(x,y)$ : reconstructed pixel value

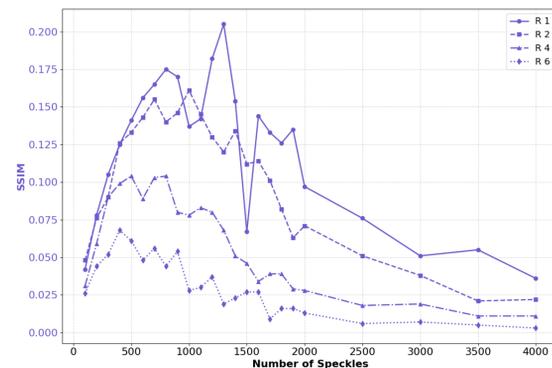
$N^2$ : total number of pixels

SSIM tells us whether the image looks structurally correct, and PSNR tells us whether the pixel values are numerically close. Using both gives a fair and balanced evaluation of reconstruction quality.

## Key Results



Effect of misalignment  $R$  on the quality of the reconstructed image with 2000 speckle patterns.



Comparison of Structural Similarity Index Measure (SSIM) values relative to the number of speckles for various parameters  $R=1, 2, 4$ , and  $6$ .

### • Even small misalignments severely degrade quality

- At  $R = 0$  (perfect alignment), both SSIM and PSNR are relatively high.
- Increasing  $R$  (shift of a few pixels) causes SSIM and PSNR to drop rapidly, and reconstructed images lose structural detail.
- **Image examples confirm metric trends**
  - With  $R = 0$  the letters are still readable,
  - At  $R \geq 2$ , structure becomes heavily distorted, and by  $R = 6$  the reconstruction is essentially lost.
- **Effect on scaling with number of speckles**
  - For  $R = 1$ , SSIM initially improves with more speckle patterns but then saturates.
  - For larger misalignments ( $R = 2, 4, 6$ ), SSIM remains low regardless of the number of speckles, meaning additional measurements cannot compensate for misalignment.

### Key Messages

- Accurate reproduction of speckle patterns is **critical for CS-SPI**.
- Even pixel-scale misalignments drastically reduce reconstruction quality, setting practical tolerance limits for experimental systems.
- Increasing the number of speckles helps only when alignment is nearly perfect.